DM898 Parameterized Algorithms
Fall 2025

Exercise Sheet 2

Complete before class on Wednesday, September 17th

Exercise 1. The Set Packing problem is defined as follows: we are given a family $\mathcal{A}$ of subsets of a universe U ($S \subseteq U$ for each $S \in \mathcal{A}$) and a number h. We want to determine if there are h sets in $\mathcal{F}$ that are pairwise disjoint.
In other words, find $S_1, \ldots, S_h \subseteq F$ with $S_i \cap S_j = \emptyset$ for all $i \neq j$.
Using dynamic programming over subsets, give an algorithm that solves Set Packing in time $2^{|U|}|\mathcal{A}|^{O(1)}$. Note that $\mathcal{F}$ could be exponentially larger than U.

Exercise 2. Show that the Set Packing problem is NP-complete. You can use that the Independent Set problem is NP-hard.

Linear programming

Exercise 3 (Solved together during class)**.** Give integer linear programming formulations for:

- Knapsack
- Clique problem
- Vertex coloring (moderately difficult)
- Steiner Tree (very difficult)

Hint: It is not easy to enforce the connectivity in Steiner tree using linear constraints. One way is to add one constraint for each $S \subseteq V$ with $\emptyset \subsetneq K \cap S \subsetneq K$. What can you say about the edges (u, v) with $u \in S$ and $v \notin S$?