DM898 Parameterized Algorithms
Fall 2025

Exercise Sheet 4

Complete before class on Monday, October 6th

Exercise 1. A 3-SAT formular consists of n Boolean variables $x_1, \ldots, x_n$ and a logical AND over m clauses, each forming a logical OR of at most 3 literals. A literal is a variable x_i or a negated variable $\neg x_i$. The 3-SAT problem asks for a given 3-SAT formular to find an assignment of variables satisfying every clause. For example consider the formular:

$$(x_1 \vee x_2 \vee \neg x_4) \wedge (\neg x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_1 \vee x_2)$$

A satisfying assignment is $x_1 = \text{false}, x_2 = \text{true}, x_3 = \text{true}, x_4 = \text{true}$. A trivial enumeration algorithm achieves a running time of $2^n \cdot (nm)^{O(1)}$. Using branching, design a $1.74^n \cdot (nm)^{O(1)}$ time algorithm.
Hints: Use a similar approach to the Closest String algorithm from the lecture.

a. *Show that you can find two assignments x, x' such that one of them differs in at most $n/2$ variables from a satisfying assignment (if one exists)*

b. *Starting with some infeasible variable assignment generate at most three alternative assignments so that one is closer to a satisfying assignment (if one exists)*

Exercise 2. The following binary linear program is derived from a Knapsack instance:

$$\begin{aligned} \max 17x_1 + 10x_2 + 25x_3 + 17x_4 \\ 5x_1 + 3x_2 + 8x_3 + 7x_4 \leq 12 \\ x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

Solve it using LP-based Branch-and-Bound.
Note that the optimal fractional solution to the LP relaxation of a Knapsack instance can be computed as follows: in non-increasing order of profit/weight select as much of each item as possible until the capacity is reached. For example, for the initial problem the optimal fractional solution is $x_1 = 1, x_2 = 1, x_3 = 0.5, x_4 = 0$.